Assignment Four: More on operations in bases other than 10
Due: 8:25am Thursday, October 6, 2016
50 points

1. The numbers below are in base 8. Perform the operation in binary.

331.57 + 2.06 (14.53 – 5.756) + (10.06 – 7.07) (16.61 + 5.1766)

Solution:

Since all numbers are in base 8 and $8 = 2^3$

We can use the grouping technique. Remember that when we had a binary we would separate it in groups of 3 for integer part from right to left and for fractional part from left to right. Then using a simple conversion table, we would figure out the equivalent values in binary. For example, to convert binary number 10100111.11001 to equivalent octal value, here is what we do:

1. Group in 3 from right to left for the whole number part, 10 100 111, then add enough zeros to the left of the left-most number if it is not already a group of three. In the above, the left-most number is 10, so we will write it as 010 to make it a group of three.
2. For the fractional part we also group them in three, but this time from left to right and if the last one is not complete, we add additional zeros. So, 11001 will become 110 01 and adding one zero, we get 110 010 which will make the final number 010100111.110010.
3. Now, using a simple conversion table such as the one below, we realize that:

In base 2	In base 8
010	2
100	4
111	7
110	6

Which will result in 247.62 in base 8.

4. The reverse is true as well. That is, we will assign a three-digit binary equivalent to each digit of base 8 value. For example, for 247.62 in base 8 we find equivalent three-digit values for 2, 4, 7, 6, and 2 and write them next to each other in the same order: 010100111.110010

DECIMAL (BASE 10)	BINARY (BASE 2)	OCTAL (BASE 8)	HEXADECIMAL (BASE 16)
0	00000	0	0
1	00001	1	1
2	00010	2	2
3	00011	3	3
4	00100	4	4
5	00101	5	5
6	00110	6	6
7	00111	7	7
8	01000	10	8
9	01001	11	9
10	01010	12	A
11	01011	13	B
12	01100	14	C
13	01101	15	D
14	01110	16	E
15	01111	17	F
16	10000	20	10

(Please note that you can print this table and bring it to the test with yourself.)

Using the same technique, we have:

Base 8	Base 2
331.57	011011001.101111
2.06	010.000110
14.53	001100.101011
5.756	101.111101110
10.06	001000.000110
7.07	111.000111
16.61	001110.110001
5.1766	101.001111110110

Performing operations (and deleting unnecessary zeros from left and right sides of the numbers):

$(16.61 + 5.1766)_8 \rightarrow$ 1110.110001 + 101.00111111011 = $(10100.00000011011)_2$

$(10.06 - 7.07)_8 \rightarrow$ 1000.00011 - 111.000111 = $(0.111111)_2$

```
     1 1 1 1 1   1 1 1 1 1
         1 0 1 . 0 0 1 1 1 1 1 1 0 1 1
 +     1 1 1 0 . 1 1 0 0 0 1
     -------------------------------------
     1 0 1 0 0 . 0 0 0 0 0 0 1 1 0 1 1
```

```
       1 0 0 0 . 0 0 0 1 1 0
 -       1 1 1 . 0 0 0 1 1 1
       1 1 1 1   1 1 1 1 1
     -------------------------
       0 0 0 0 . 1 1 1 1 1 1
```

Performing multiplication:

$(16.61 + 5.1766)_8 * (10.06 - 7.07)_8 = (10100.00000011011)_2 * (0.111111)_2$

First we multiply two numbers without considering the fractional point. Then we count the total number of places after the fractional point for both (first number 11 and second number 6, totaling 17) and we enter the fraction point after 17^{th} place from the right to left.

						1	0	1	0	0	0	0	0	0	0	0	1	1	0	1	1
					x											1	1	1	1	1	1
														1	1	1					
												1	1	1	1	1	1				
1	1	1	1	1	1						1	1	1	1	1	1	1	1	1		
						1	0	1	0	0	0	0	0	0	0	0	1	1	0	1	1
					1	0	1	0	0	0	0	0	0	0	0	1	1	0	1	1	
				1	0	1	0	0	0	0	0	0	0	0	1	1	0	1	1		
			1	0	1	0	0	0	0	0	0	0	0	1	1	0	1	1			
		1	0	1	0	0	0	0	0	0	0	0	1	1	0	1	1				
	1	0	1	0	0	0	0	0	0	0	0	1	1	0	1	1					
1	0	0	1	1	1	0	1	1	0	0	1	1	0	1	0	1	0	0	1	0	1

1 0 0 1 1 1 . 0 1 1 0 0 1 1 0 1 0 1 0 0 1 0 1

Now, the next group:

$2.06_8 * (14.53 - 5.756)_8 \rightarrow 010.000110_2 * (001100.101011 - 101.111101110)_2$

Performing subtraction:

	1	1	0	0	.	1	0	1	0	1	1	0	0
-		1	0	1	.	1	1	1	1	0	1	1	1
	1	1	1	1		1	1	1		1	1	1	
	0	1	1	0	.	1	0	1	1	0	1	0	1

Now multiplication of $10.00011_2 * 110.10110101_2$

						1	1	0	1	0	1	1	0	1	0	1
				x						1	0	0	0	0	1	1
				1	1											
		1	1	1	1	1	1	1	1	1						
						1	1	0	1	0	1	1	0	1	0	1
					1	1	0	1	0	1	1	0	1	0	1	
				0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0			
		0	0	0	0	0	0	0	0	0	0	0				
	0	0	0	0	0	0	0	0	0	0	0					
1	1	0	1	0	1	1	0	1	0	1						
1	1	1	0	0	0	1	0	1	0	1	0	1	1	1	1	1

1 1 1 0 . 0 0 1 0 1 0 1 0 1 1 1 1 1

Finally, adding three values:

331.57 + 2.06 (14.53 – 5.756) + (10.06 – 7.07) (16.61 + 5.1766)

In binary:

11011001.101111 + 1110.0010101011111 + 100111.0110011010100101

											1														
1	1	1	1	1	1	1	1	1		1	1	1	1	1	1		1	1	1						
	1	1	0	1	1	0	0	1	.	1	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0
			1	0	0	1	1	1	.	0	1	1	0	0	1	1	0	1	0	1	0	0	1	0	1
					1	1	1	0	.	0	0	1	0	1	0	1	0	1	1	1	1	1	0	0	0
1	0	0	0	0	1	1	1	1	.	0	1	0	1	1	1	0	1	1	0	0	1	1	1	0	1

2. The numbers below are in different bases. Find the result in binary.

$2331_4 + 2f06_{16}\ (100111010110_2 - 30724_8) + (1006_8 - 134033_4)\ (16c6e_{16} + 1948_{10})$

Partial Solution:

Due to a typo in the statement of the problem. This problem will not be graded and all 50 points for the assignment is allocated to problem 1. Those who have done some steps of this problem will receive points as bonus point.

The difference between this problem and the previous one is that in here the numbers are presented in different bases whereas in previous problem they were all presented in base 8. We are going to convert all numbers to base 2 and then perform the operation. In here we do not have any fractional values involved which makes it a bit easier.

To convert values in bases 4, 8, and 16 we can use grouping technique. For base 10 we have to use the standard method explained in previous solutions. To convert from base 4 to binary we use grouping of 2. To convert from base 8 to binary we use grouping of 3. To convert from base 18 to binary we use grouping of 4.

$2331_4 \rightarrow 10111101_2$

$2f06_{16} \rightarrow 0010111100000110_2$

$30724_8 \rightarrow 011000111010100_2$

$1006_8 \rightarrow 001000000110_2$

$134033_4 \rightarrow$ There is a typo in this value. There cannot be a 4 in base 4 numbers. This number was originally base 10 value.

$16c6e_{16} \rightarrow 00010110110001101110_2$

With all the numbers converted into binary the rest of operations are similar to the previous problem.