Examples - Geometric Gradient Series Factors

EXAMPLE 2.11

A coal-fired power plant has upgraded an emission control valve. The modification costs only $8000 and is expected to last 6 years with a $200 salvage value. The maintenance cost is expected to be high at $1700 the first year, increasing by 11% per year thereafter. Determine the equivalent present worth of the modification and maintenance cost by hand and by spreadsheet at 8% per year.

Solution by Hand

The cash flow diagram (Figure 2–22) shows the salvage value as a positive cash flow and all
costs as negative. Use Equation [2.35] for g ≠ i to calculate Pg. Total PT is the sum of three
present worth components.

$$\begin{aligned} P_T &= -8000 - P_g + 200(P/F,8\%,6) \\ &= -8000 - 1700\left[\frac{1-(1.11/1.08)^6}{0.08-0.11}\right] + 200(P/F,8\%,6) \\ &= -8000 - 1700(5.9559) + 126 = \$-17{,}999 \end{aligned}$$

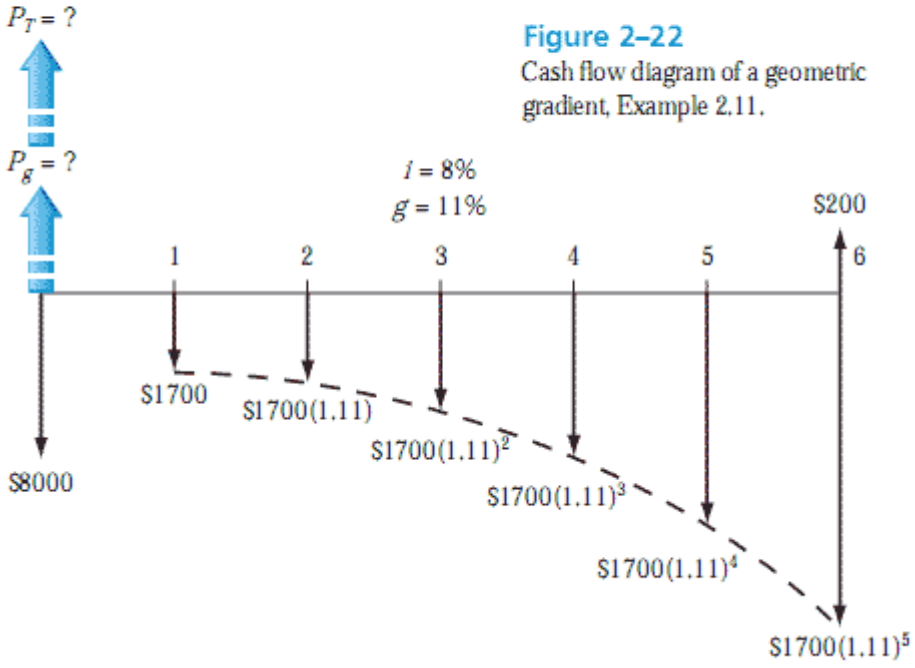


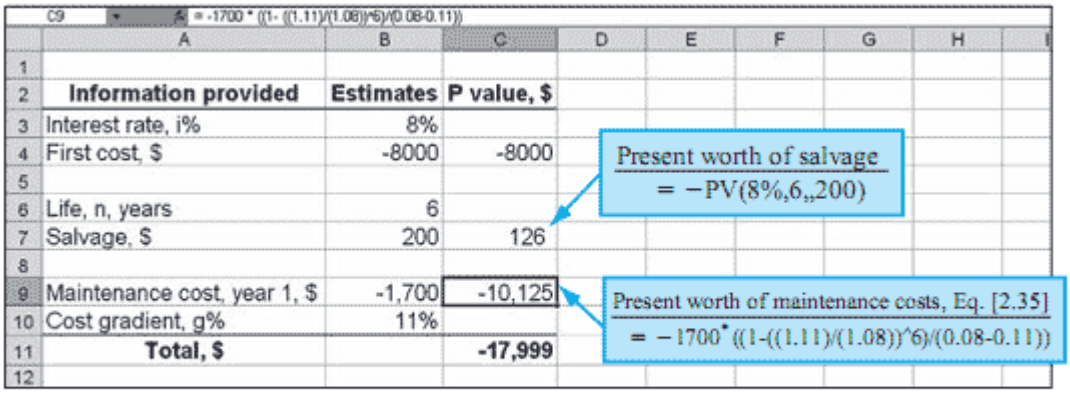
Figure 2–22
Cash flow diagram of a geometric gradient, Example 2.11.

Solution by Spreadsheet

Figure 2–23 details the spreadsheet operations to find the geometric gradient present worth Pg and total present worth PT. To obtain PT = $- 17,999, three components are summed—first cost, present worth of estimated salvage in year 6, and Pg . Cell tags detail the relations for the second and third components; the first cost occurs at time 0.

Comment

The relation that calculates the (P /A,g,i%,n) factor is rather complex, as shown in the cell tag and formula bar for C9. If this factor is used repeatedly, it is worthwhile using cell reference formatting so that A1 , i , g, and n values can be changed and the correct value is always obtained. Try to write the relation for cell C9 in this format.



	A	B	C	D	E	F	G	H
1								
2	**Information provided**	**Estimates**	**P value, $**					
3	Interest rate, i%	8%						
4	First cost, $	-8000	-8000					
5								
6	Life, n, years	6						
7	Salvage, $	200	126					
8								
9	Maintenance cost, year 1, $	-1,700	-10,125					
10	Cost gradient, g%	11%						
11	**Total, $**		**-17,999**					
12								

Figure 2–23
Geometric gradient and total present worth calculated via spreadsheet, Example 2.11.

EXAMPLE 2.12 - The Cement Factory Case

Now, let's go back to the proposed Houston American Cement plant in Georgia. The revenue series estimate of $50 million annually is quite optimistic, especially since there are many other cement product plants operating in Florida and Georgia on the same limestone deposit. (The website for the HAC plant shows where they are located currently; it is clear that keen competition will be present.) Therefore, it is important to be sensitive in our analysis to possibly declining and increasing revenue series, depending upon the longer-term success of the plant's marketing, quality, and reputation. Assume that revenue may start at $50 million by the end of the first year, but then decreases geometrically by 12% per year through year 5. Deter- mine the present worth and future worth equivalents of all revenues during this 5-year time frame at the same rate used previously that is, 10% per year.

Solution

The cash flow diagram appears much like Figure 2–21 b , except that the arrows go up for revenues. In year 1, A1 $50 M and revenues decrease in year 5 to

$$A_1(1-g)^{n-1} = 50\text{ M}(1-0.12)^{5-1} = 50\text{ M}(0.88)^4 = \$29.98\text{ M}$$

First, we determine Pg in year 0 using Eq. [2.35] with i 0.10 and g 0.12, then we calculate F in year 5. In $1 million units,

$$P_g = 50\left[\frac{1-\left(\frac{0.88}{1.10}\right)^5}{0.10-(-0.12)}\right] = 50[3.0560]$$

$$= \$152.80$$

$$F = 152.80(F/P,10\%,5) = 152.80(1.6105)$$

$$= \$246.08$$

This means that the decreasing revenue stream has a 5-year future equivalent worth of $246.080 M. If you look back to Example 2.6, we determined that the F in year 5 for the uniform revenue series of $50 M annually is $305.255 M. In conclusion, the 12% declining geometric gradient has lowered the future worth of revenue by $59.175 M, which is a sizable amount from the perspective of the owners of Houston American Cement of North America, Inc.

Source:

http://engineeringandeconomicanalysis.blogspot.com/2014/03/examples-geometric-gradient-series.html