IEGR 350: Engineering Economy
Summer 2017
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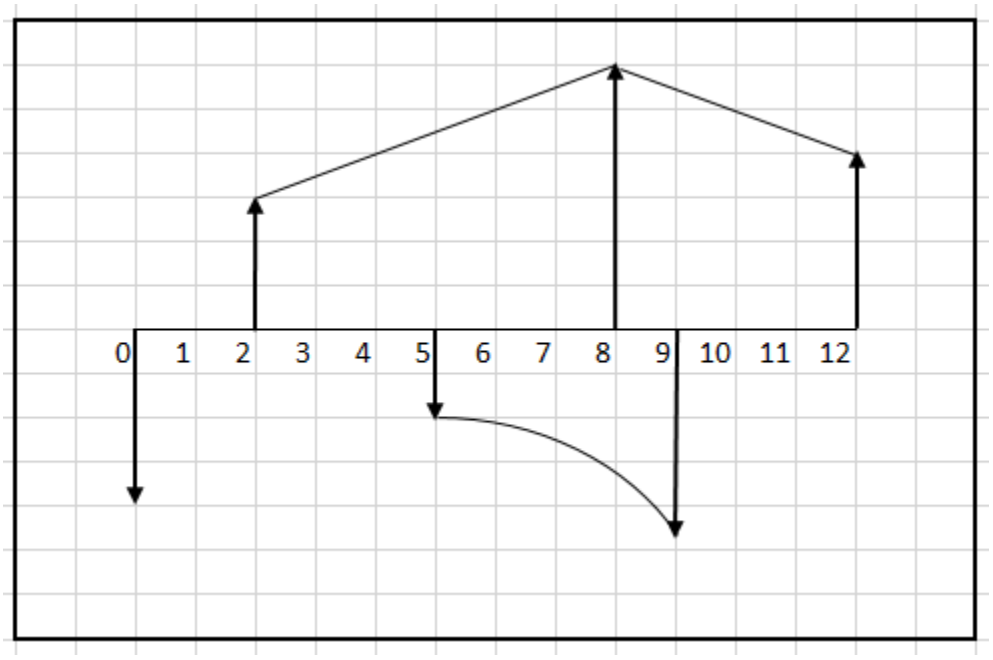
Test 2

Time: 100 Minutes

There are three books available for you to get the factors. Write your formulations and then move to front/back of the class to use the tables. After recording the values, go back to your desk and do the calculations.

PROBLEM 1: (35 Points)

Find the rate of return for the project below. Use tables and interpolation. Each grid is $550. All numbers are at exact grids except the investment at year 9, which is $2,500.



Solution:

The goal of the problem is to find the rate of return. There are no restriction provided other than the use of tables (for factors) and interpolation (which means, no use of formulas). Therefore, the easiest approach is to use individual end-of-year transactions.

Inflow Transactions:

Year	0	1	2	3	4	5	6	7	8	9	10	11	12
Amount	0	0	1650	1925	2200	2475	2750	3025	3300	3025	2750	2475	2200

Calculations:

Between 2 and 8 we have an arithmetic gradient series with G1 (2-0G1, 3-1G1, 4-2G1, 5-3G1, 6-4G1, 7-5G1 and 8-6G1).

G1 = (3300 – 1650) / 6 = 275.

Therefore, beginning at the end of year 2 with 1650, end of year 3 payment is:

1650 + 275 = 1925, and so on.

Similar calculations for gradient G2 between 8 and 12:

(2200 - 3300) / 4 = - 275

Therefore, beginning at the end of year 8 with 3300, end of year 9 payment is:

3300 - 275 = 3025, and so on.

outflow Transactions:

Year	0	1	2	3	4	5	6	7	8	9	10	11	12
Amount	2200	0	0	0	0	1100	1350.61	1658.31	2036.12	2500	0	0	0

Calculations:

Between 5 and 9 we have a geometric gradient series with g:

5 $1100(1+g)^0$

6 $1100(1+g)^1$

7 $1100(1+g)^2$

8 $1100(1+g)^3$

9 $1100(1+g)^4$

At 9 the value is 2500, therefore, $1100(1+g)^4 = 2500$ from which

$(1+g)^4 = 2500/1100$ and, $(1+g) = 1.2278$

Net Cashflow:

Year	Expenditure	Income	Net Cashflow
0	-$2,000.00	$0.00	-$2,000.00
1	$0.00	$0.00	$0.00
2	$0.00	$1,650.00	$1,650.00
3	$0.00	$1,925.00	$1,925.00
4	$0.00	$2,200.00	$2,200.00
5	-$1,100.00	$2,475.00	$1,375.00
6	-$1,350.61	$2,750.00	$1,399.39
7	-$1,658.31	$3,025.00	$1,366.69
8	-$2,036.12	$3,300.00	$1,263.88
9	-$2,500.00	$3,025.00	$525.00
10	$0.00	$2,750.00	$2,750.00
11	$0.00	$2,475.00	$2,475.00
12	$0.00	$2,200.00	$2,200.00

To begin guessing for ROR, find the total net cash flow (i.e. at 0% ROR) which is 17k+ (very large number pointing to large ROR). Select 40% initially.

Int. Rate		0.4		0.5		0.6	
N	Net Cashflow	(P/F, I, N)	PW	(P/F, I, N)	PW	(P/F, I, N)	PW
0	-$2,000.00	1.0000	-$2,000.00	1.0000	-$2,000.00	1.0000	-$2,000.00
1	$0.00	0.7143	$0.00	0.6667	$0.00	0.6250	$0.00
2	$1,650.00	0.5102	$841.84	0.4444	$733.33	0.3906	$644.53
3	$1,925.00	0.3644	$701.53	0.2963	$570.37	0.2441	$469.97
4	$2,200.00	0.2603	$572.68	0.1975	$434.57	0.1526	$335.69
5	$1,375.00	0.1859	$255.66	0.1317	$181.07	0.0954	$131.13
6	$1,399.39	0.1328	$185.85	0.0878	$122.85	0.0596	$83.41
7	$1,366.69	0.0949	$129.65	0.0585	$79.99	0.0373	$50.91
8	$1,263.88	0.0678	$85.64	0.0390	$49.31	0.0233	$29.43
9	$525.00	0.0484	$25.41	0.0260	$13.66	0.0146	$7.64
10	$2,750.00	0.0346	$95.07	0.0173	$47.69	0.0091	$25.01
11	$2,475.00	0.0247	$61.12	0.0116	$28.61	0.0057	$14.07
12	$2,200.00	0.0176	$38.80	0.0077	$16.96	0.0036	$7.82
PW	$17,129.96		$993.25		$278.42		-$200.39

Therefore, the real ROR is between 50% and 60%. Perform interpolation:

For 10% difference from 50% to 60%, we have $400.81 reduction in present worth [278.42-(-200.39)], so for 1% difference, we have $40.08 reduction. What percentage reduction do we need from 278.42 to get to 0.00?

278.42/40.08 = 6.95 which results in ROR of 56.95% (the real ROR is 55.35%).

Grading Rubric:

Whether you used individual transactions or series, you had to calculate the values of G1 and G2 (5 points) and g (5 points). Writing correct present worth of the series (15 point -- 5 points each); guessing trial ROR, calculating its present worth and indicating the next trial ROR (10 points); finding the solution through interpolation (10 bonus points)

PROBLEM 2: (30 Points)

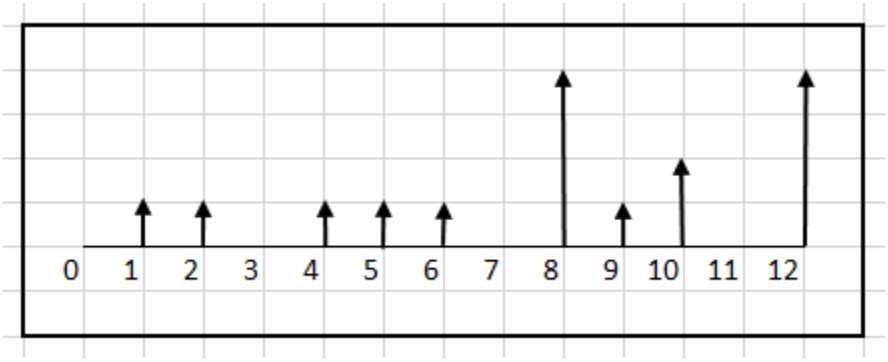
Sixteen transactions ($100 each) have occurred during a one-year project. Find the present worth of the project for different cases below:
1. MC - monthly compounding for duration of the project
2. WC - weekly compounding for duration of the project
3. CC – Continuous compounding for duration of the project.
All transactions are at the end of mentioned days. Use APR of 9%.

Day	of week	of month
1	1	1
3	6	2
2	14	4
4	19	5
5	26	6
2	33	8
6	33	8
1	35	8
3	35	8
7	35	9
4	40	10
6	42	10
2	50	12
3	50	12
4	50	12
5	52	12

Solution:

When considering monthly compounding, all transactions within a month are considered, end of the month transactions. For example, the four transactions during month of 12 are all added up to create one transaction at the end of month 12 (100+100+100+100=400). Applying the same approach, we will have the cash flow diagram presented below.

Month	Transaction
0	0
1	100
2	100
3	0
4	100
5	100
6	100
7	0
8	400
9	100
10	200
11	0
12	400



APR = 9%

Monthly interest rate = 0.09/12 = 0.75%

PW = 100(P/F,0.75%,1)+ 100(P/F,0.75%,2)+ 100(P/F,0.75%,4)+ 100(P/F,0.75%,5)+
100(P/F, 0.75%,6)+ 400(P/F,0.75%,8)+ 100(P/F,0.75%,9)+
200(P/F,0.75%,10)+ 400(P/F,0.75%,12)

PW = 100(0.9926)+ 100(0.9852)+ 100(0.9706)+ 100(0.9633)+ 100(0.9562)+
400(0.9420)+ 100(0.9350)+ 200(0.9280)+ 400(0.9142) = $1,508.36

Using similar approach for weekly compounding we get the following cash flow.

Week	Transaction
0	0
1	100
6	100
14	100
19	100
26	100
33	200
35	300
40	100
42	100
50	300
52	100

APR = 9%

Weekly interest rate = 0.09/52 = 0.173%

PW = 100(P/F,0.17%,1)+ 100(P/F,0.17%,6)+ 100(P/F,0.17%,14)+

100(P/F,0.17%,19)+ 100(P/F, 0.17%,26)+ 200(P/F,0.17%,33)+

300(P/F,0.17%,35)+ 100(P/F,0.17%,40)+ 100(P/F,0.17%,42)+

300(P/F,0.17%,50)+ 100(P/F,0.17%,52) = $1,512.92

Grading Rubric:

MC and WC calculations each had 15 points. CC calculations were dropped from the grading. Those who attempted and solved it would receive up to 10 bonus points.

PROBLEM 3: (35 Points)

A company is considering three projects with project lives of 5, 6 and 12 years. Transactions are provided for each project in the table below.

	Transactions		
Year	Alternative 1	Alternative 2	Alternative 3
0	-3000	-4750	-1500
1	2500	2050	0
2	2000	2050	250
3	1500	2050	750
4	1000	2050	1250
5	500	2050	1750
6		1050	2250
7			1000
8			1000
9			1000
10			1000
11			1000
12			600

The company uses 15% MARR compounding annually.
Which project should be selected? All projects can be repeated with the same assumptions.

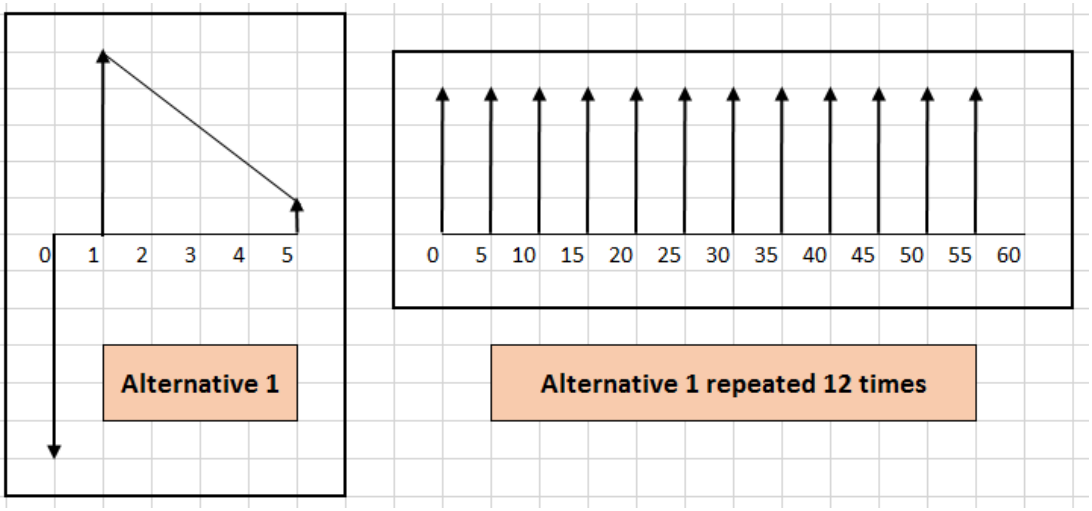
Solution:

Since, projects do not have the same lifetime, we need to find the LCM and repeat the projects accordingly.

LCM (5, 6, 12) = 60

Therefore, repeat Alternative-1, 12 times, Alternative-2, 10 times, and Alternative-3, 5 times. Let us calculate the present worth of each one.

Alternative-1: Alternative-1 consists on one initial capital investment and a series of receipts decreasing by an arithmetic gradient of $500 each year.



PW (Alt-1) = - 3000 + 2500 (P/A, 15%, 5) – 500 (P/G, 15%, 5) =

- 3000 + 2500 (3.3522) – 500 (5.775) = $2493

Now, repeat this alternative 12 times, which is equivalent of repeating a single receipt of $2493 in years 0, 5, 10 … 50 and 55. Note that there is no receipt at year 60 because the alternative-1 during the last 5 years is represented by the single equivalent at year 55. Therefore, 60-year equivalent present worth is:

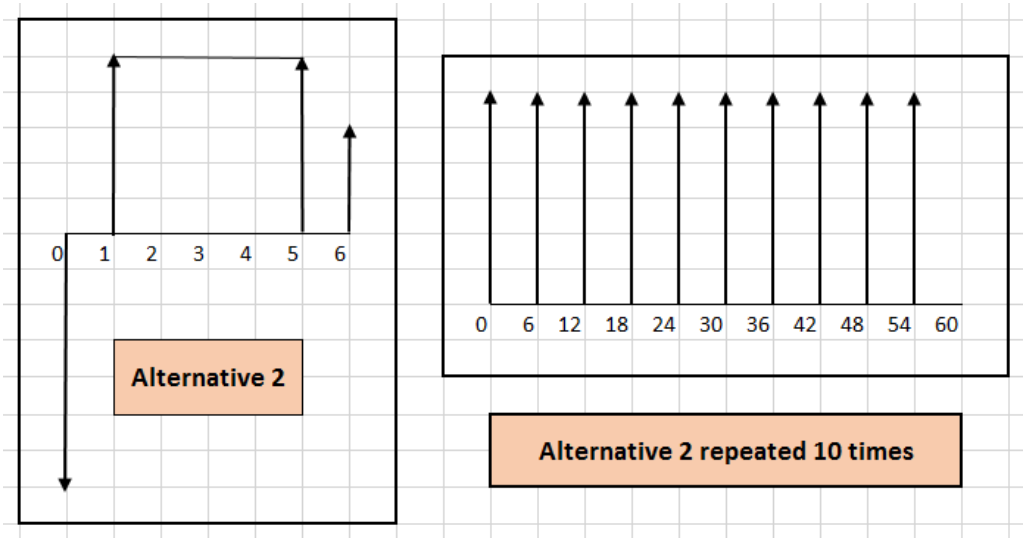
PW (Alt-1) = 2493 [1 + (P/F, 15%, 5) + (P/F, 15%, 10) + … + (P/F, 15%, 55)] =

$2493\ [1 + 1.15^{-5} + 1.15^{-10} + \ldots + 1.15^{-55}] = 2493\ (1.9883) = \$4956.87$

Alternative-2: This alternative consists of one initial capital investment and a series of uniform receipts ($2050), except the last year ($1050). We can either adjust the last one to be the same amount by adding and subtracting $1000 to it at the same time, or just leave it as it is.

PW (Alt-2) = - 4750 + 2050 (P/A, 15%, 5) + 1050 (P/F, 15%, 6) =

- 4750 + 2050 (3.3522) + 1050 (0.4323) = $2575.92



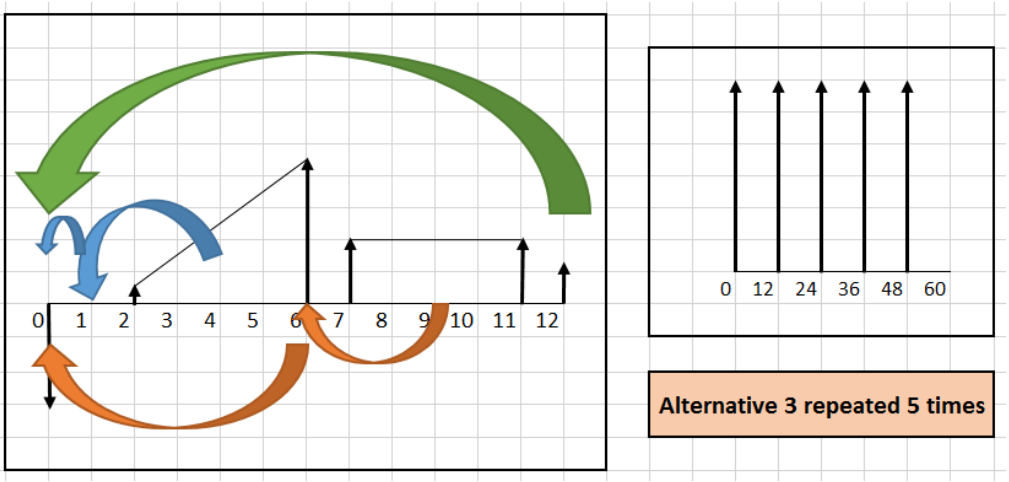
Now, repeat this alternative 10 times, which is equivalent of repeating a single receipt of $4084.42 in years 0, 6, 12 … 48 and 54. Note that there is no receipt at year 60 because the alternative-2 during the last 6 years is represented by the single equivalent at year 54. Therefore, 60-year equivalent present worth is:

PW (Alt-2) = 2575.92 [1 + (P/F, 15%, 6) + (P/F, 15%, 12) + … + (P/F, 15%, 54)] =

$$2575.92\,[1 + 1.15^{-6} + 1.15^{-12} + \ldots + 1.15^{-54}] = 2575.92\,(1.7612) =$$

$4536.72

Alternative-3: This alternative consists of one initial capital investment and two different series, an arithmetic gradient series (A=250, G=500) and a uniform series ($1000), except the last year ($600).



PW (Alt-3) = - 1500 + [250 (P/A, 15%, 5) + 500 (P/G, 15%, 5)] (P/F, 15%, 1) +

1000 (P/A, 15%, 5) (P/F, 15%, 6) + 600 (P/F, 15%, 12) =

- 1500 + [250 (3.3522) + 500 (5.7751)] (0.8696) +

1000 (3.3522) (0.4323) + 600 (0.1869) = $3301.08

Now, repeat this alternative 5 times, which is equivalent of repeating a single receipt of $2328.30 in years 0, 12, 24, 36, 48 and 60. Note that there is no receipt at year 60 because the alternative-3 during the last 12 years has already been accounted for by the single equivalent at year 48. Therefore, 60-year equivalent present worth is:

PW (Alt-3) = 3301.08 [1+(P/F,15%,12)+(P/F,15%,24)+(P/F,15%,24)+(P/F,15%,54)] =

3301.08 [1 + 1.15^{-12} + 1.15^{-24} + 1.15^{-36} + 1.15^{-48}] = 3301.08 (1.2296) =

$4058.98

Comparing the PW of three alternatives, we conclude Alternative-1 as the best choice (PW=$4956.87).

Grading Rubric:

5 points each for individual present worth calculations of each alternatives; 5 points each for repeated individual present worth calculations of each alternatives; 5 points for correct selection with accurate numbers.