IEGR 410: Simulation
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The probability density function (pdf) of a Triangular distribution given as:

$$f(x) = \begin{cases} \dfrac{2(x-a)}{(m-a)(b-a)} & for\ a \le x \le m \\ \dfrac{2(b-x)}{(b-m)(b-a)} & for\ m \le x \le b \\ 0 & Otherwise \end{cases}$$

For Triangular (4.63, 7.05, 13.94), then a=4.63, m=7.05, and b=13.94. Calculating the denominators values:

(m - a) (b - a) = (7.05 – 4.63) (13.94 – 4.63) = 2.42(9.31) = 22.530

(b – m) (b - a) = (13.94 – 7.05) (13.94 – 4.63) = 6.89(9.31) = 64.146

2/22.530 = 0.08877

2/64.146 = 0.03118

So,

$$f(x) = \begin{cases} 0.08877(x-4.63) & for\ 4.63 \le x \le 7.05 \\ 0.03118(13.94-x) & for\ 7.05 \le x \le 13.94 \\ 0 & Otherwise \end{cases}$$

To generate random numbers from this distribution using the technique discussed in the handout

1. Let k =(m - a) / (b – a) = 2.42 / 9.31 = 0.259936

2. Generate u from U(0, 1), let say u = 0.2381

3. Since, u is smaller than k, we use the formula y = $(k.u)^{0.5}$ = = $(0.259936*0.2381)^{0.5}$ = 0.248778

4. The generated random value from Triangular distribution is:

x = a + $(b – a)^{y}$ = 4.63 + $(13.94 – 4.63)^{0.248778}$ = 6.3720

Using Microsoft Excel simplifies the process in an efficient manner. Here is an example of how I set up my Excel sheet.

The Y value calculation came from the following conditional statement:

IF (D2<=B11, SQRT (D2*B11),(1-SQRT((1-D2)*(1-B11))))

b-a	9.31	0.258	0.259	6.411
b-m	6.89	0.541	0.417	7.167
m-a	2.42	0.048	0.112	5.913
(m - a)(b - a)	22.5302	0.277	0.268	6.450
(b – m)(b - a)	64.1459	0.455	0.365	6.888
2/(m - a)(b - a)	0.08877	0.439	0.356	6.842
2/(b – m)(b - a)	0.031179	0.494	0.388	7.008
k	0.259936	0.577	0.440	7.301
u	0.2381	0.359	0.311	6.632
y	0.248778	0.552	0.424	7.206
X	6.372022	0.420	0.345	6.788
1-u	0.7619	0.063	0.128	5.960
1-k	0.740064	0.662	0.500	7.682
(1-u)(1-k)	0.563855	0.291	0.276	6.480
SQRT[(1-u)(1-k)]	0.750903	0.506	0.395	7.046
1-SQRT[(1-u)(1-k)]	0.249097	0.307	0.284	6.513

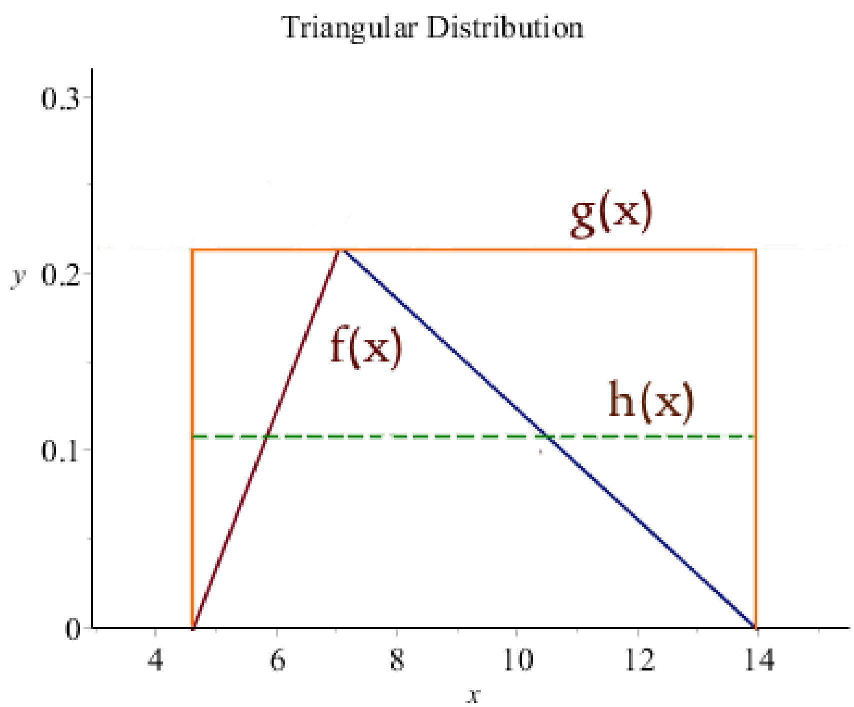
Column D contains
Uniform (0, 1)
values from RV-bank

Column F contains
TRIA (4.63, 7.05, 13.94)
values

To generate random numbers from the same triangular distribution using acceptance-rejection method we need to find the maximum value of f(x) which occur when x = m. Replacing the value of x in the pdf, f(m) = 0.214823. Therefore we define the majorizing function as g(x) = 0.214823 for the range of (a, b). Since, g(x) is not a legitimate probability distribution (because the area underneath the curve is not equal to 1) we are going to make sure that is true by setting a ratio of g(x)/area under g(x). This function, h(x), is a legitimate uniform distribution between 4.63 and 13.94.

h(x) = 0.214823/0.214823*(13.94 – 4.63) = 1/9.31 = 0.107411

Now, we can generate a random number U (4.63, 13.94) simply by generating a u from U (0,1) and calculating X* = a + (b – a) u = 4.63 + 9.31 u. We also need to find the ratio of f(x*)/g(x*) that we need for last step of acceptance-rejection method. The last step is generating a new u from U (0, 1) and comparing it to that ratio for acceptance of rejection of X*.



The following Excel sheet setup is a sample of set up for this problem.

U (0, 1)	x*	f(x*)/g(x*)	U (0, 1)	ACC/REJ X*
0.554	8.072	0.852	0.042	8.072
0.466	7.460	0.940	0.891	7.460
0.716	9.572	0.634	0.508	9.572
0.492	7.628	0.916	0.457	7.628
0.705	9.450	0.652	0.067	9.450
0.289	6.534	0.787	0.894	REJECT
0.271	6.461	0.756	0.335	6.461
0.338	6.755	0.878	0.477	6.755
0.732	9.751	0.608	0.366	9.751
0.501	7.685	0.908	0.704	7.685
0.506	7.719	0.903	0.387	7.719
0.626	8.669	0.765	0.407	8.669
0.831	11.014	0.425	0.625	REJECT
0.084	5.836	0.498	0.646	REJECT
0.643	8.824	0.743	0.676	8.824
0.089	5.849	0.504	0.312	5.849
0.808	10.703	0.470	0.507	REJECT
0.576	8.248	0.826	0.193	8.248