IEGR 350: Engineering Economy
Spring 2016
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Assignment 7
PROBLEM:
A company is considering 3 projects with project lives of 6, 8 and 10 years. Transactions are provided for each project in the table below.

	Transactions		
Year	Alternative 1	Alternative 2	Alternative 3
0	-2250	-2750	-3250
1	1400	1050	2200
2	1300	1050	1950
3	1200	1050	1700
4	1100	1050	1450
5	1000	1050	1200
6	500	1050	950
7		1050	-100
8		800	300
9			300
10			900

The company uses 12% MARR compounding annually except for year 5 of each project where the compounding rates are different (MC for Alternative 1, WC for Alternative 2, and CC for Alternative 3). Which project should be selected? All projects can be repeated with the same assumptions

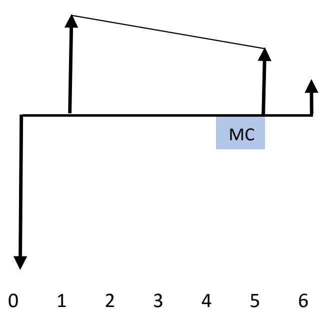
Since, projects have different life time numbers we need to find the LCM of 6, 8, 10.
6 = 2x3, 8 = 2x2x2, 10 = 2x5. Therefore, LCM = 2x2x2x3x5 = 120.
Thus we need to repeat Alternatinve 1 twenty times (120/6 = 20), Alternatinve 2 fifteen times (120/8 = 15), and Alternatinve 3 twelve times (120/10 = 12).
Before, writing the cashflow for 120 years, we find the equivalent present worth of each project and then set up the 120 years cashflow.

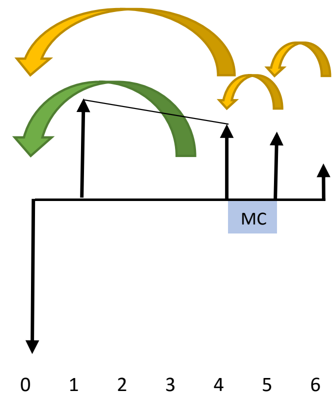
First, let's calculate the effective interest rates for different periods that have different compounding assumptions than yearly.

Effective Interest rate calculation:
Nominal: 12%, (MC) monthly compounding
$r = (1 + i/n)^{n} - 1 = (1+0.12/12)^{12} - 1 =$
1.01268 - 1 = 0.1268 or 12.68%

Effective Interest rate calculation:
Nominal: 8%, (CC) continuous compounding
$r = e^{i} - 1 = (2.718)^{0.12} - 1 =$
1.1275 - 1 = 0.1275 or 12.75%

Effective Interest rate calculation:
Nominal: 12%, (WC) weekly compounding
$r = (1 + i/n)^{n} - 1 = (1+0.12/52)^{52} - 1 =$
1.1273 - 1 = 0.1273 or 12.73%

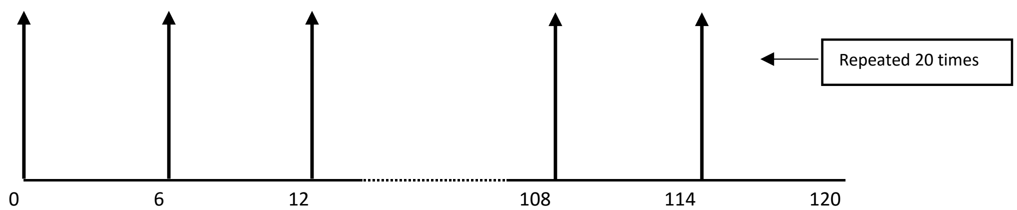


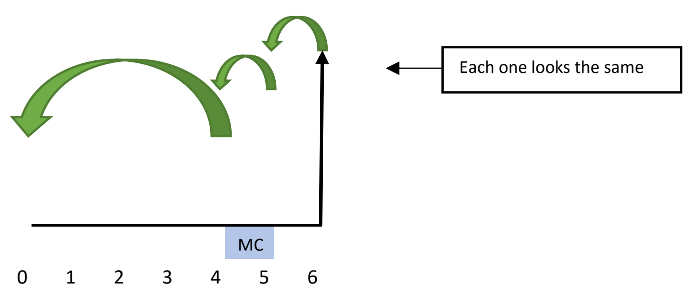


Alternative 1 present worth calculations:

PW1 = - 2250 + 1400 (P/A, 12%, 4) - 100 (P/G, 12%, 4) + [500 (P/F, 12%, 1) + 1000] (P/F, 12.68%, 1)(P/F, 12%, 4) =

PW1 = - 2250 + 1400 (3.0373) - 100 (4.1273) + [500 (0.8929) + 1000] (0.8875)(0.6355) = $2405.30





Similar conversions for each unit (example is for finding present worth of payment at year 114 in year 108) :

P = 2405.30 (P/F, 12%, 1) (P/F, 12.68%, 1) (P/F, 12%, 4) = 2405.30 (0.8929) (0.8875)(0.6355) = $2405.30 (0.5036)

Now this value will be added to the $2405.30 which is already at year 108

P(@108) = $2405.30 + $2405.30 (0.5036) = 2405.30 [1 + (0.5036)]

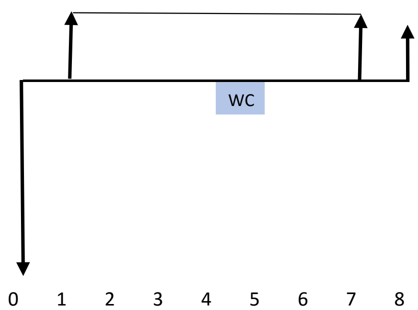
To move this value to year 102 we need to multiply this value by (0.5036) and add the results to the value 2405.30 already at year 102. The result after processing is : 2405.30 [1 + (0.5036) + (0.5036)(0.5036)] =

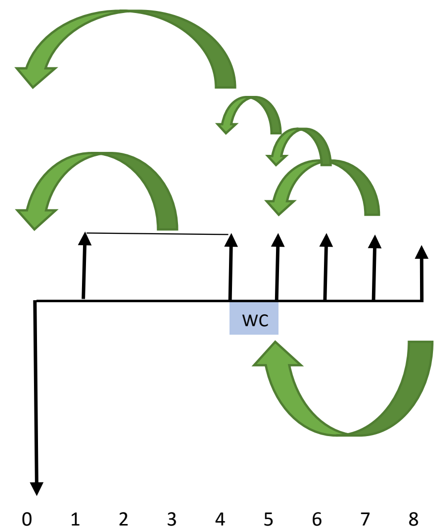
$2405.30\,[\,(0.5036)^{0} + (0.5036)^{1} + (0.5036)^{2}]$ which is for 2 moves. In general for X number of moves we will have :

$2405.30\,[\,(0.5036)^{0} + (0.5036)^{1} + (0.5036)^{2} + \ldots + (0.5036)^{X-1} + (0.5036)^{X}] = 2405.30\,\{\,[1 - (0.5036)^{X+1}]/(1-0.5036)\}$

In our case we need to move 19 times, so:

$PW = 2405.30\,\{\,[1 - (0.5036)^{18}]/(1-0.5036)\} = \$4845.48$

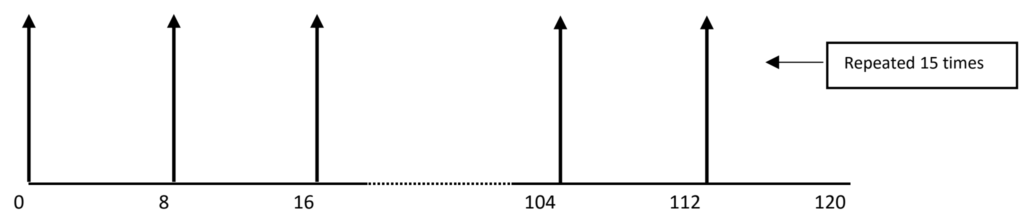


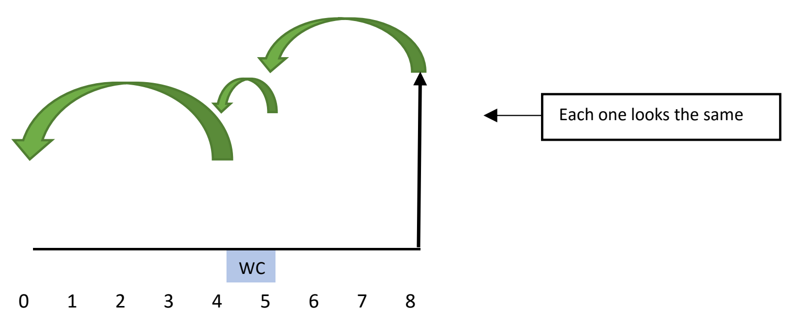


Alternative 2 present worth calculations:

PW2 = - 2750 + 1050 (P/A, 12%, 4) + 800 (P/F, 12%, 3) (P/F, 12.73%, 1)(P/F, 12%, 4) + 1050 (P/F, 12%, 2) (P/F, 12.73%, 1)(P/F, 12%, 4) + 1050 (P/F, 12%, 1) (P/F, 12.73%, 1)(P/F, 12%, 4) =

PW2 = - 2750 + 1050 (3.0373) + 800 (0.7118) (0.8871)(0.6355) + 1050 (0.7972) (0.8871)(0.6355) + 1050 (0.8929) (0.8871)(0.6355) = $1760.63





Similar conversions for each unit (example is for finding present worth of payment at year 112 in year 104) :

P = 1760.63 (P/F, 12%, 3) (P/F, 12.73%, 1) (P/F, 12%, 4) = 1760.63 (0.7118) (0.8871)(0.6355) = $706.50

Now this value will be added to the $1760.63 which is already at year 104

P(@108) = $1760.63 + $1760.63 (0.4013) = 1760.63 [1 + (0.4013)]

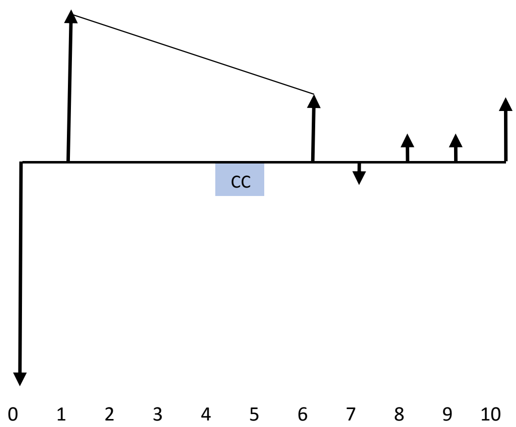
To move this value to year 96 we need to multiply this value by (0.4013) and add the results to the value1760.63 already at year 96. The result after processing is : 1760.63 [1 + (0.4013) + (0.4013)(0.4013)] =

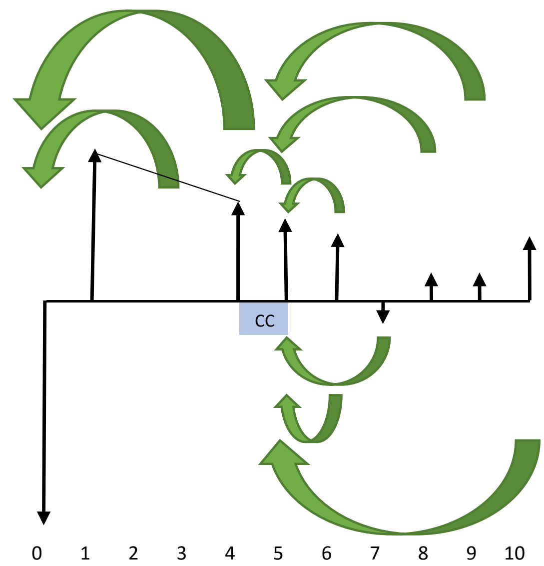
1760.63 [$(0.4013)^0 + (0.4013)^1 + (0.4013)^2$] which is for 2 moves. In general for X number of moves we will have :

1760.63 [$(0.4013)^0 + (0.4013)^1 + (0.4013)^2 + \ldots + (0.4013)^{X-1} + (0.4013)^X$] = 1760.63 { [1 - $(0.4013)^{X+1}$]/(1-0.4013)}

In our case we need to move 14 times, so:

PW = 1760.63 { [1 - $(0.4013)^{14}$]/(1-0.4013)} = $2940.75

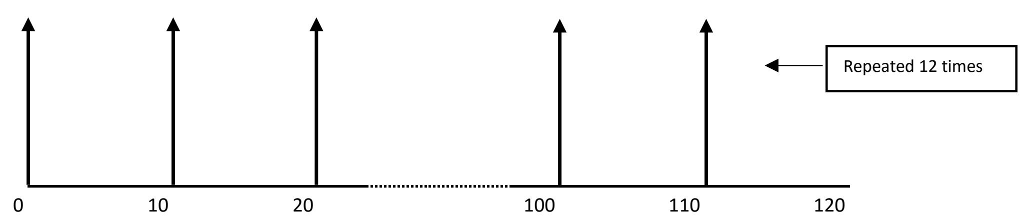


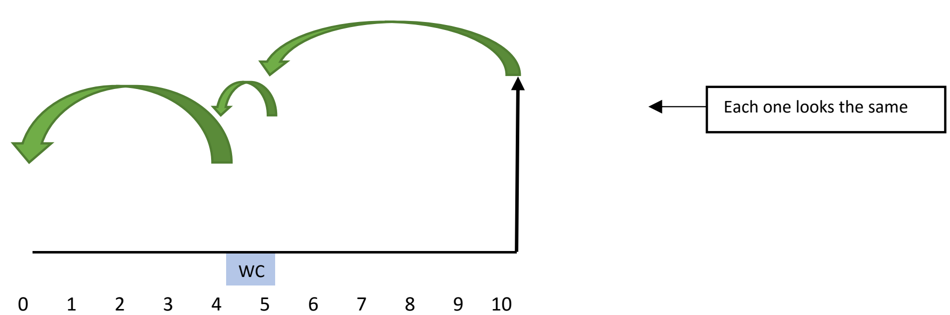


Alternative 3 present worth calculations:

PW3 = - 3250 + 2200 (P/A, 12%, 4) + 250 (P/G, 12%, 4) + 1200 (P/F, 12.75%, 1)(P/F, 12%, 4) +
950 (P/F, 12%, 1)(P/F, 12.75%, 1)(P/F, 12%, 4) - 100 (P/F, 12%, 2)(P/F, 12.75%, 1)(P/F, 12%, 4) +
300 (P/F, 12%, 3) (P/F, 12.75%, 1)(P/F, 12%, 4) + 300 (P/F, 12%, 4) (P/F, 12.75%, 1)(P/F, 12%, 4) +
900 (P/F, 12%, 5) (P/F, 12.75%, 1)(P/F, 12%, 4)

PW3 = - 3250 + 2200 (3.0373) + 250 (4.1273) + 1200 (0.8869)(0.6355) + 950 (0.8929)(0.8869)(0.6355)
- 100 (0.7972)(0.8869)(0.6355) + 300 (0.7118) (0.8869)(0.6355) + 300 (0.6355) (0.8869)(0.6355) +
900 (0.5674) (0.8869)(0.6355) = $6089.03





Similar conversions for each unit (example is for finding present worth of payment at year 110 in year 100) :

P = 6089.03 (P/F, 12%, 5) (P/F, 12.75%, 1) (P/F, 12%, 4) =6089.03 (0.5674) (0.8869)(0.6355) = $1947.28

Now this value will be added to the $6089.03 which is already at year 100

P(@100) = $6089.03 + $6089.03 (0.3198) = 6089.03 [1 + (0.3198)]

To move this value to year 90 we need to multiply this value by (0.3198) and add the results to the value6089.03 already at year 90. The result after processing is : 6089.03 [1 + (0.3198) + (0.3198)(0.3198)] =

6089.03 [$(0.3198)^0 + (0.3198)^1 + (0.3198)^2$] which is for 2 moves. In general for X number of moves we will have :

6089.03 [$(0.3198)^0 + (0.3198)^1 + (0.3198)^2 + \ldots + (0.3198)^{X-1} + (0.3198)^X$] = 6089.03 { [1 - $(0.3198)^{X+1}$]/(1-0.3198)}

In our case we need to move 11 times, so:

PW = 6089.03 { [1 - $(0.3198)^{11}$]/(1-0.3198)} = $8951.79 making alternative 3 the best choice.