

Test 3

100 Points (Time: 90:00 Minutes + 10 Minutes if needed)

Instruction for all problems: Show your work. No round down or up, use 2 decimals for dollar values and 4 decimals for factors.

Question 1: (30 points)

The data below show two patterns of inflation that are exactly the opposite of each other over a 20-year time period.

- If each machine costs \$10,000 in year 0 and they both increase in cost exactly in accordance with the inflation rate, how much will each machine cost at the end of year 20?
- What is the average inflation rate over the time period for machine A (that is, what single inflation rate would result in the same final cost for machine A)?
- In which years will machine A cost more than machine B?

Year	Machine A, %	Machine B, %
1	10	2
2	10	2
3	2	10
4	2	10
5	10	2
6	10	2
7	2	10
8	2	10
.	.	.
.	.	.
.	.	.
19	2	10
20	2	10

Solution:

$$\begin{aligned} \text{(a) Cost, year 20: machine A} &= 10,000(1.10)(1.10)(1.02)(1.02)\dots(1.02) \\ &= \$31,617.58 \end{aligned}$$

$$\begin{aligned} \text{Cost, year 20: machine B} &= 10,000(1.02)(1.02)(1.10)(1.10)\dots(1.10) \\ &= \$31,617.58 \end{aligned}$$

The cost is the same.

$$\begin{aligned} \text{(b) } 10,000(1+f)^{20} &= 31,617.58 \\ (1+f)^{20} &= 3.1618 \\ 20[\log(1+f)] &= \log 3.1628 \\ \log(1+f) &= 0.0250 \\ 1+f &= 10^{0.025} \\ 1+f &= 1.05925 \\ f &= 5.925\% \end{aligned}$$

$$\begin{aligned} \text{(c) Year 1: Machine A cost} &= 10,000(1.10) = \$11,000 \\ \text{Machine B cost} &= 10,000(1.02) = \$10,200 \end{aligned}$$

$$\begin{aligned} \text{Year 2: Machine A cost} &= 11,000(1.10) = \$12,100 \\ \text{Machine B cost} &= 10,200(1.02) = \$10,404 \end{aligned}$$

$$\begin{aligned} \text{Year 3: Machine A cost} &= 12,100(1.02) = \$12,342 \\ \text{Machine B cost} &= 10,404(1.10) = \$11,444.40 \end{aligned}$$

$$\begin{aligned} \text{Year 4: Machine A cost} &= 12,342(1.02) = \$12,588.84 \\ \text{Machine B cost} &= 11,444.40(1.10) = \$12,588.84 \end{aligned}$$

Machine A will cost more than machine B in all years except years 4, 8, 12, 16, and 20.

Question 2: (20 points) Select one of the options and solve

OPTION 1: Exactly 10 years ago, Boyditch Professional Associates purchased \$100,000 in depreciable assets with an estimated salvage of \$10,000. For tax depreciation, the SL method with $n = 10$ years was used, but for book depreciation, Boyditch applied the DDB method with $n = 7$ years and neglected the salvage estimate. The company sold the assets today for \$12,500.

- (a) Compare the sales price today with the book values using the SL and DDB methods.
 (b) If the salvage of \$12,500 had been estimated exactly 10 years ago, determine the depreciation for each method in year 10.

i (a) SL: $BV_{10} = \$10,000$ by definition

DDB: Determine if the implied $S < \$10,000$ with $d = 2/7 = 0.2857$

$$\begin{aligned} BV_{10} &= BV_7 = 100,000(0.7143)^7 \\ &= \$9488 \end{aligned}$$

Both salvage values are less than the market value of \$12,500

(b) SL: $D_{10} = (100,000 - 12,500)/10 = \8750 per year

DDB: $D_{10} = 0$, since $n = 7$ years

OPTION 2: An investor who purchased a \$10,000 mortgage bond today paid only \$6000 for it. The bond coupon rate is 8% per year, payable quarterly, and the maturity date is 18 years from the year of issuance. Because the bond is in default, it will pay no dividend for the next 2 years. If the bond dividend is in fact paid for the following 5 years (after the 2 years) and the investor then sells the bond for \$7000, what rate of return will be realized (a) per quarter and (b) per year (nominal)?

$$\begin{aligned} I &= 10,000(0.08)/4 \\ &= \$200 \text{ per quarter} \end{aligned}$$

$$(a) 0 = -6000 + 200(P/A, i, 20)(P/F, i, 8) + 7000(P/F, i, 28)$$

Solve for i by trial and error

$$i = 2.55\% \text{ per quarter}$$

$$\begin{aligned} (b) \text{ Nominal annual } i &= 0.0255(4) \\ &= 10.2\% \text{ per year, compounded quarterly} \end{aligned}$$

Question 3: (30 points)

On a student loan, you have received \$10,000 twice a year on January 1st and August 1st of each year for the last 9 semesters. Rate is 8% compounded monthly. Upon graduation, you start paying back your loan. What should your monthly payments be to pay back your loan in the same amount of months it took you to graduate? You began your education in Fall semester and graduated in December.

Solution:

You began your education in Fall semester and graduated in December, so you received 5 payments in End of December (or first day of January) and 4 payments at the end of July (or first of August). To find the present worth (at time of graduation) you can find the future worth of each individual payment at time of graduation. You need to remember that periods are in months and number of periods (n) must be in months. Thus the payment you received for Fall semester before graduation has n=4. Since payments are equal we just need to add the (F/P, i, n) factors and multiply the results by \$10,000. Also note that we need to calculate the rate of return per period.

$$i = 0.08/12 = 0.0066 = 0.66\%$$

$$\text{Sum} = (F/P, 0.66\%, 52) + (F/P, 0.66\%, 48) + (F/P, 0.66\%, 40) + (F/P, 0.66\%, 36) + (F/P, 0.66\%, 28) + (F/P, 0.66\%, 20) + (F/P, 0.66\%, 16) + (F/P, 0.66\%, 8) + (F/P, 0.66\%, 4)$$

$$\text{Sum} = 1.4078 + 1.3712 + 1.3009 + 1.2672 + 1.2022 + 1.1406 + 1.1109 + 1.0540 + 1.0266 = 10.8819$$

$$\text{PW} = \$10,000 (10.8819) = \$108,819.00$$

This value must be repaid over the same number of months (52) in equal payment.

$$\$108,819 = A (P/A, 0.66\%, 52)$$

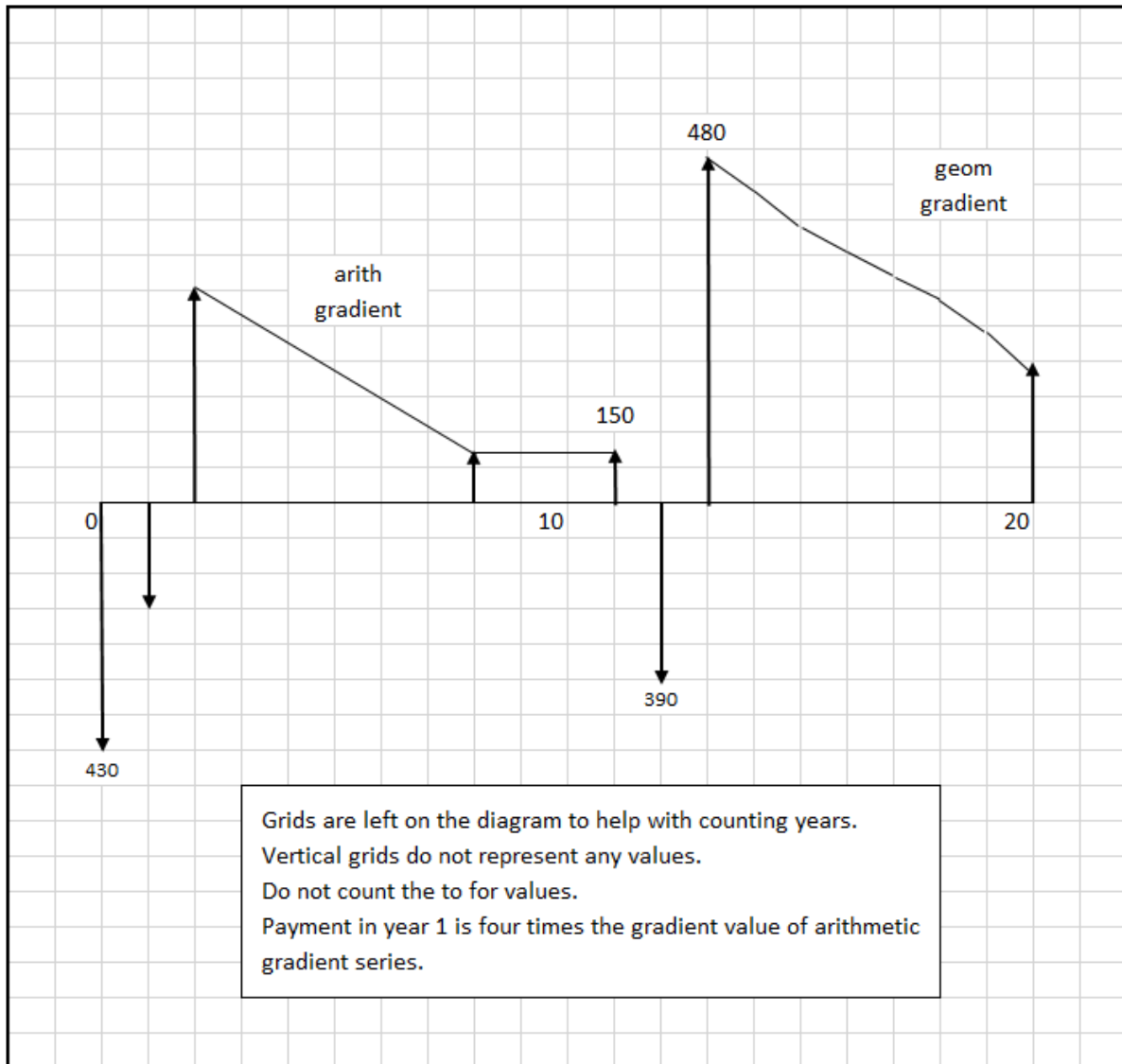
$$\left(\frac{P}{A}, 0.66\%, 52\right) = \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] = \left[\frac{(1.0066)^{52} - 1}{0.0066(1.0066)^{52}}\right] = \left[\frac{0.4078}{0.0092}\right] = 43.8944$$

$$A = \$108,819/43.8944 = \$2479.10$$

Reality check: So, overall, you received 9 (\$10,000) = \$90,000 and you are paying back 52 (2479.10) = \$128,913.58. Welcome to real life.

Question 4: (20 points)

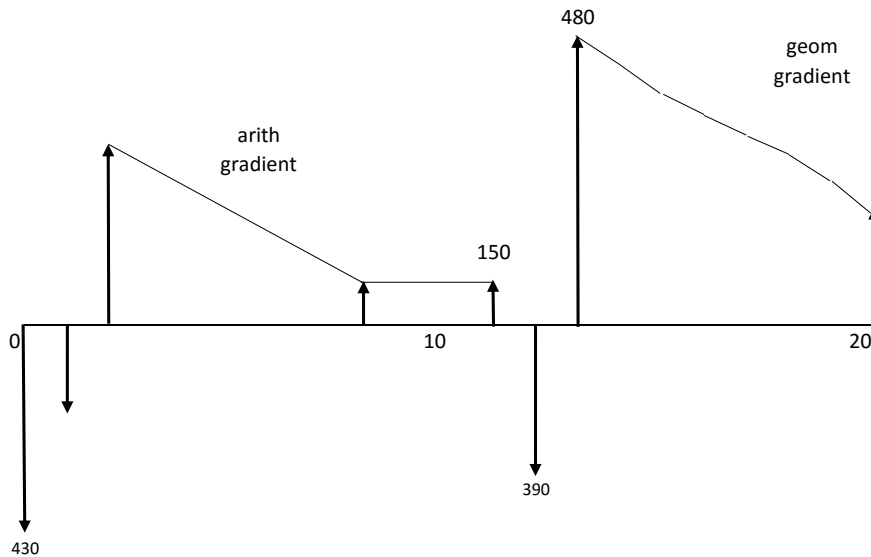
Develop a relationship between the values of the gradients for geometric and arithmetic gradient series for the project to have 16% rate of return (annual compounding).



Develop a relationship between the values of the gradients for geometric and arithmetic gradient series for the project to have 16% rate of return (annual compounding).

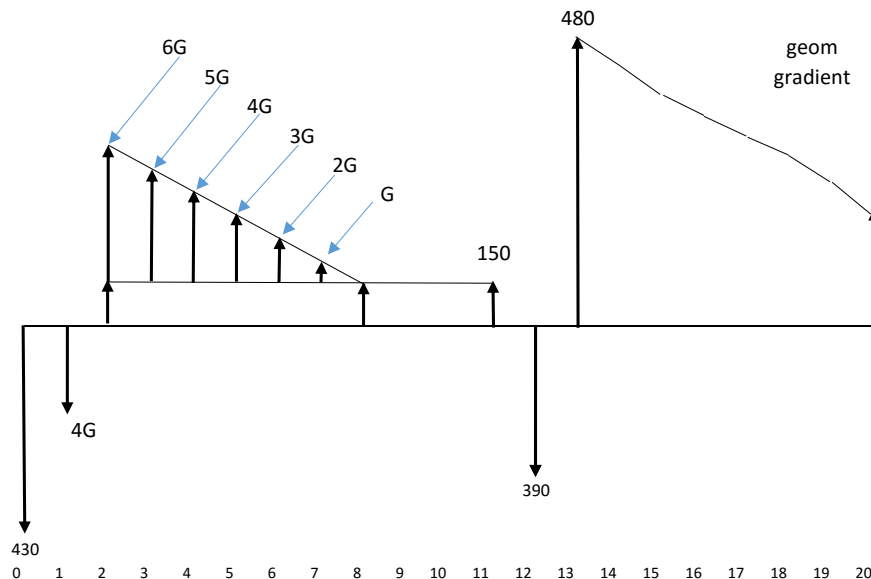
Grids are left on the diagram to help with counting years. Size of vertical grids do not represent any specific value. Do not count them for values.

Payment in year 1 is four times the gradient value of arithmetic gradient series.



We will use G to represent the gradient for the arithmetic gradient series and g for the gradient of geometric gradient series. All values are given but to calculate the initial value of the decreasing geometric series we have to do some initial work.

4.8332	625.0053
0.8621	3.4484
0.1685	65.715
4.0386	24.2316
9.761	14.4706
	12.4751
	80.88
	9.026704
	1120.72
	65.715
	495.715
	724.98
	625.0053
	15.2049
	13.10814
	129.2903
	9.659741



Now we can write the present worth of expenditures and set them equal to the income at the or return of 16%.

Finding present value of arithmetic gradient series at 0:

$$PW1 = -430 - 4G(P/F, 16\%, 1) - 390(P/F, 16\%, 12) = -430 - 4G(0.8621) - 390(0.1685) = -495.715 - 3.4484G$$

$$PW2 = [150(P/A, 16\%, 10) + 6G(P/A, 16\%, 7) - G(P/G, 16\%, 7)](P/F, 16\%, 1) + PW(\text{geom series})$$

$$PW2 = [150(4.8332) + 6G(4.0386) - G(9.7610)](0.8621) + PW(\text{geom series})$$

$$PW2 = (724.98 + 24.2316G - 9.7610G)(0.8621) + PW(\text{geom series})$$

$$PW2 = 625.005 + 13.1081G + PW(\text{geom series})$$

geometric series decreasing by g with A1=480, i=16% and n=8

$$PW(\text{geometric series}) = A1 \left\{ \left[1 - \left(\frac{1+g}{1+i} \right)^n \right] / (i-g) \right\} = 480 \left\{ \left[1 - \left(\frac{1+g}{1+0.16} \right)^n \right] / (0.16-g) \right\}$$

$$PW1 + PW2 = -495.715 - 3.4484G + 625.005 + 13.1081G + 480 \left\{ \left[1 - \left(\frac{1+g}{1+0.16} \right)^n \right] / (0.16-g) \right\} = 0$$

$$129.29 + 9.6597G + 480 \left\{ \left[1 - \left(\frac{1+g}{1+0.16} \right)^n \right] / (0.16-g) \right\} = 0$$